SUBLIMATION OF ICE IN COARSELY DISPERSED ROCKS SUBJECT TO QUASIISOTHERMAL CONDITIONS

A. Z. Volynets, É. D. Ershov,
I. A. Komarov, and V. K. Safonov

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The results of a theoretical and experimental investigation into the sublimation of ice in coarsely dispersed rocks are presented.

Interest in the processes underlying the sublimation of ice in dispersed rocks has recently increased sharply. This has occurred because of the development of work on various underground engineering constructions in the zone of perpetual frost, such as underground stores for liquefied natural gases, gas tunnel conduits, underground cold storage (refrigerators), etc. This process is also of no small importance in establishing the thermal conditions and humidity of soils.

The results of an experimental investigation into the sublimation of ice in coarsely dispersed rocks [1, 2] revealed the mechanism and qualitative laws of the process. Thus, when moist soil interacts with a vapor—air medium at negative temperatures the soil loses moisture and a dry zone is formed. A characteristic feature of the process in coarsely dispersed soils is the fact that the transfer of moisture in the dry zone occurs mainly in the form of vapor. Heat and mass transfer take place by a purely molecular process: heat conduction and diffusion. The extremely low-temperature gradients occurring during the sublimation of ice enable us to examine the process under quasiisothermal conditions. The intensity of the process is determined and limited, not by heat inflow to the zone of the phase transitions, but by mass (vapor) transfer in the dry zone. In order to determine the law governing the motion of the sublimation front we may accordingly use the equation of mass transfer, taking the gradient of the partial pressure of the water vapor as the motive force.

These combined concepts enable us to pass to a mathematical model of the process in question by making use of the method of conjugate boundary-value problems with a moving boundary. We shall consider the problem for two cases of practical importance: that in which the diffusion resistance between the surface of the soil and the vapor—gas medium (ambient) may be neglected and that in which it may not.

1. We have a semifinite, coarsely dispersed solid with an initial moisture content w_n. The moisture in the material is distributed uniformly, with a degree of pore occupation equal to unity, and it exists in a frozen state, i.e., as ice. The amount of unfrozen moisture may be neglected. The temperature of the solid and gas—vapor ambient is T_0. The partial pressure of the water vapor above the ice situated in the pores of the coarsely dispersed rocks equals the partial pressure of saturated water vapor over ice, p_n.

At the initial instant of time $\tau > 0$ a partial water-vapor pressure corresponding to the relative humidity of the vapor—gas flow is established instantaneously at the solid—ambient interface and held constant during the whole process of ice sublimation.

Under these conditions the field of partial vapor pressures in the material is described by the equation

$$\frac{\partial p}{\partial \tau} = k_p \frac{\partial^2 p}{\partial x^2}; \quad (\tau > 0;\ 0 \leqslant x \leqslant \xi); \tag{1}$$

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with boundary conditions

$$\tau = 0; \quad p = p_{\mathrm{n}}; \tag{2}$$

$$\tau > 0; \quad x = 0; \quad p = p_{\mathrm{p}}; \tag{3}$$

$$\tau > 0; \quad x = \xi; \quad p = p_{\mathrm{n}}; \tag{4}$$

$$\tau > 0; \quad x = \xi; \quad \frac{k_{\mathrm{p}} \partial p}{R_{\mathrm{p}} T_0 \partial x} = \gamma_0 w_{\mathrm{n}} \frac{d\xi}{d\tau}. \tag{5}$$

The solution to this problem takes the form

$$p = p_{\mathrm{p}} + \frac{p_{\mathrm{n}} - p_{\mathrm{p}}}{\operatorname{erf}\left(\dfrac{\xi}{2\sqrt{k_{\mathrm{p}}\tau}}\right)} \operatorname{erf}\left(\frac{x}{2\sqrt{k_{\mathrm{p}}\tau}}\right). \tag{6}$$

Since condition (5) should be satisfied for all τ, the following equation determining the phase-transition boundary should also be valid:

$$\xi = 2\omega \sqrt{k_{\mathrm{p}}\tau}, \tag{7}$$

where ω is a numerical factor, which is constant for the process under consideration. Substituting (6) into (5) and differentiating, we obtain an equation for ω

$$\omega \exp(\omega^2) \operatorname{erf}(\omega) = \frac{p_{\mathrm{n}} - p_{\mathrm{p}}}{\gamma_0 w_{\mathrm{n}} R_{\mathrm{p}} T_0 \sqrt{\pi}}. \tag{8}$$

This equation may be solved either graphically or by successive approximations.

The sublimation intensity is defined by

$$\frac{k_{\mathrm{p}}}{R_{\mathrm{p}} T_0} \cdot \frac{\partial p}{\partial x} = I_{\mathrm{s}} \quad \text{for} \quad x = 0. \tag{9}$$

Differentiating Eq. (6), substituting the resultant expression into (9), and transforming, we obtain

$$I_{\mathrm{s}} = \frac{k_{\mathrm{p}} (p_{\mathrm{n}} - p_{\mathrm{p}})}{\sqrt{\pi k_{\mathrm{p}} \tau}\, R_{\mathrm{p}} T_0 \operatorname{erf}(\omega)}. \tag{10}$$

In view of the fact that substantial difficulties arise in determining ω from Eq. (8), it is important to have a simplified formula for calculating this quantity; this may be obtained if we expand the functions $\operatorname{erf}(\omega)$ and $\exp(\omega^2)$ in series. Taking the first terms of these expansions [3], we obtain

$$\omega = \sqrt{\frac{p_{\mathrm{n}} - p_{\mathrm{p}}}{2\gamma_0 w_{\mathrm{n}} R_{\mathrm{p}} T_0}}. \tag{11}$$

Calculation shows that the value of ω determined from Eq. (11) corresponds to the value obtained from Eq. (8) to within less than 1%. The position of the phase-transition boundary and the amount of sublimed moisture are, respectively, given by

$$\xi = \sqrt{\frac{2k_{\mathrm{p}}(p_{\mathrm{n}} - p_{\mathrm{p}})}{R_{\mathrm{n}} T_0 \gamma_0 w_{\mathrm{n}}} \tau} \tag{12}$$

and

$$m = \sqrt{\frac{2k_{\mathrm{p}} (p_{\mathrm{n}} - p_{\mathrm{p}})}{R_{\mathrm{p}} T_0} \gamma_0 w_{\mathrm{n}} \tau}. \tag{13}$$

Equations (12) and (13) may be obtained from simple physical considerations if we assume that the distribution of the partial pressures approximately coincides with the distribution corresponding to a steady mass flow. For boundary conditions (3) and (4) we have the following expressions for p:

$$p = p_{\mathrm{p}} + (p_{\mathrm{n}} - p_{\mathrm{p}}) \frac{x}{\xi}. \tag{14}$$

Differentiating Eq. (14) and substituting into (5), we obtain

$$k_{\mathrm{p}} \frac{p_{\mathrm{n}} - p_{\mathrm{p}}}{R_{\mathrm{p}} T_0 \xi} = \gamma_0 w_{\mathrm{n}} \frac{d\xi}{d\tau}. \tag{15}$$

Separating the variables and integrating the latter expression, we obtain Eq. (12). The intensity of sublimation will be given by

$$I_{\mathrm{s}} = \sqrt{\frac{k_{\mathrm{p}} (p_{\mathrm{n}} - p_{\mathrm{p}}) \gamma_0 w_{\mathrm{n}}}{R_{\mathrm{p}} T_0 2\tau}}. \tag{16}$$

It should be remembered that Eqs. (12) and (16) are only valid when $(p_{\mathrm{n}} - p_{\mathrm{p}})/\gamma_0 w_{\mathrm{n}} R_{\mathrm{p}} T_0 \ll 1$. This relationship is almost always valid for dispersed rocks under natural conditions and forms the basis for our quasi-steady-state approach to the solution of the problem at hand.

The experimental verification of the resultant relationships was carried out in an experimental laboratory apparatus [2], supplemented by a copper heat exchanger maintaining a specified temperature on the lower surface of the samples. As coolant we used antifreeze (ethylene glycol) cooled in a TS-16 thermostat placed in a cold chamber. The experiments were carried out on sand samples of various fractions. The diffusion coefficient of the water vapor in the sand was determined by a steady-state method. The sand sample fraction was placed in a vertical cylindrical cassette directly on the ice surface. Diffusion of the vapor was achieved by virtue of the drop in partial pressure between the surface of the ice and the vapor–gas flow, which was kept constant over the whole experiment. Between the surface of the sand and the vapor–gas flow was an air gap 2 cm high; the vapor transfer coefficient in this was determined by a calibration experiment. The value of k_{p} was determined from Eq. (9). Analysis of the experimental data regarding the diffusion coefficient of the vapor in the sand was based on the equation

$$k_{\mathrm{p}} = \alpha s D_{\mathrm{a}}. \tag{17}$$

Experiments show that the tortuosity coefficient for the sand fractions studied (0.25-1 mm) varied from 0.4 to 0.6, respectively, for a volumetric proportion of free pores s = 1. The diffusion coefficient of the vapor in air was determined from the equation

$$D_{\mathrm{a}} = D_0 \left(\frac{T_0}{T_{\mathrm{n}}} \right)^2,$$

where

$$D_0 = 0.22 \cdot 10^{-4} \ \mathrm{m^2/sec}; \quad T_{\mathrm{n}} = 273.16\ {}^\circ\mathrm{K}.$$

This equation is in satisfactory agreement with the data of other authors who have studied the diffusion of water vapor [1, 4] and other gases such as acetone and CO_2 in sand [5].

Figure 1 shows the experimental and calculated values of the sublimation intensity and the thickness of the dried sand zone at various moments of time. We see that the values of I_{s} and ξ derived from Eqs. (12) and (16) agree satisfactorily with experimental data, so vindicating the quasi-steady-state approach to this problem.

2. The field of partial pressures and the boundary conditions are described in this case by Eqs. (1)-(2) and (4)-(5). Instead of Eq. (3), at the boundary x = 0 we write the condition

$$\tau > 0; \quad x = 0; \quad k_{\mathrm{p}} \frac{\partial p}{\partial x} = \beta [p_{\mathrm{p}}(\tau) - p_{\mathrm{av}}]. \tag{18}$$

An exact solution cannot be obtained for this problem, and we accordingly seek approximate solutions. To this end we write the problem in new variables,

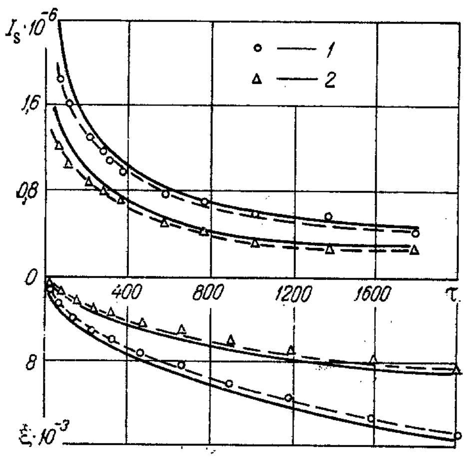

Fig. 1

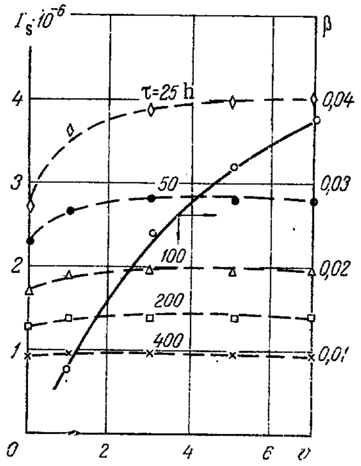

Fig. 2

Fig. 1. Change in the intensity of ice sublimation and thickness of the dry sand zone in relation to the time of the process: 1) n = 0.4; t = –5°C; φ = 70%; v = 10 m/sec; 2) n = 0.45; t = –12°C; φ = 80%; v = 10 m/sec (continuous line, calculation; broken line, experiment). I_s, kg/m^2·sec; τ, h; ε, m.

Fig. 2. Effect of the velocity of the air flow on the mass-transfer coefficient and the intensity of ice sublimation in sand at various instants of time. I_s, kg/m^2·sec; β and v, m/sec.

$$\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial X^2}\quad (t>0;\ \ 1\leqslant X\leqslant H); \tag{19}$$

$$\psi=\psi_0=1;\quad H=1;\quad t=1; \tag{20}$$

$$\frac{\partial\psi}{\partial X}=\psi_p;\quad X=1;\quad t>1; \tag{21}$$

$$\psi=\psi_0;\quad \frac{\partial\psi}{\partial X}=\frac{1}{2\omega_1^2}\cdot\frac{dH}{dt};\quad X=H;\quad t>0, \tag{22}$$

where

$$X=\frac{x+\Delta}{\Delta};\quad H=\frac{\xi+\Delta}{\Delta};\quad \Delta=\frac{k_p}{\beta};\quad \psi=\frac{p-p_{av}}{p_n-p_{av}};$$

$$t=\frac{k_p\tau+\Delta^2}{\Delta^2};\quad \omega_1^2=\frac{p_n-p_{av}}{2\gamma_0 w_n R_p T_0};\quad \psi_n=\frac{p_p-p_{av}}{p_n-p_{av}}.$$

Taking a linear partial vapor-pressure distribution law (the validity of the quasi-steady-state approach was indicated when considering the previous problem), we obtain

$$\psi=\frac{X}{H}. \tag{23}$$

The partial pressure on the soil surface is determined from the condition (21) in the form

$$\psi_n=\frac{1}{H}, \tag{24}$$

while the law of motion of the phase interface is given by Eq. (22):

$$H=\sqrt{4\omega_1^2(t-1)+1}\ , \tag{25}$$

or in the original variables

$$\xi = \sqrt{4k_p\omega_1^2\tau + \Delta^2} - \Delta. \tag{26}$$

Differentiating Eq. (26) and multiplying by $\gamma_0 w_n$ we obtain an expression for the sublimation intensity:

$$I_s = \frac{2k_p\omega_1^2\gamma_0 w_n}{\sqrt{4k_p\omega_1^2\tau + \Delta^2}}. \tag{27}$$

If other diffusion resistances comparable with the resistance of the dry soil exist (if, for example, the surface of the soil is covered with a layer of dry material having a porosity differing from that of the main soil), the quantity Δ is calculated from the equation

$$\Delta = k_p\left(\frac{1}{\beta} + \sum_{i=1}^{n} R_i\right). \tag{28}$$

For a long enough period of the process, the quantity Δ^2 becomes much smaller than $4k_p\omega_1^2\tau$. In this case Δ may be neglected and Eq. (27) transforms into (16) (Fig. 2).

The mass-transfer coefficient is determined from a critical equation derived by analogy with the thermal case [2]:

$$\mathrm{Nu} = A(\mathrm{Re})^n, \tag{29}$$

where $A = 0.028$, $n = 0.8$ in the range $4.8 \cdot 10^3 < \mathrm{Re} < 5 \cdot 10^4$.

Verification of Eq. (29) was carried out for the sublimation of pure ice at various airflow velocities (Fig. 2); the calculated results were in excellent agreement with experiment.

NOTATION

p, partial pressure of water vapor, $N \cdot m^{-2}$; x, coordinate, m; τ, time, sec; K_p, diffusion coefficient of water vapor in the material, $m^2 \cdot sec^{-1}$; R_p, gas constant, $J \cdot kg^{-1} \cdot {}^\circ C^{-1}$; ξ, thickness of dry zone, m; γ_0, specific gravity of soil framework, $kg \cdot m^{-3}$; p_{av}, partial pressure of water vapor in the gas flow, $N \cdot m^{-2}$; I_s, intensity of ice sublimation, $kg \cdot m^{-2} \cdot sec^{-1}$; m, amount of sublimed moisture, $kg \cdot m^{-2}$; α, tortuosity coefficient; n, free porosity; D_a, diffusion coefficient of water vapor in gas, $m^2 \cdot sec^{-1}$; β, mass-transfer coefficient, $m \cdot sec^{-1}$; R_i, diffusion resistance of i-th layer, $m^{-1} \cdot sec$; v, velocity, $m \cdot sec^{-1}$; t, temperature, °C; Nu, Re, Nusselt and Reynolds numbers; P_p, partial pressure of the water vapor at the gas–material interface, $N \cdot m^{-2}$.

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